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On Supra I-open Sets and Supra I-continuous functions

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Abstract— In this paper we introduce and investigate a new class of sets and functions between topological spaces called supra I-open sets and supra I-continuous functions respectively.

Keywords— Supra I-open set; Supra I-continuous functions; Supra topological spaces.

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1 INTRODUCTION

IN 1983, A.S.Mashhour et al. [6] introduced the supra topological spaces and studied S-continuous maps and S*-continuous maps. In 1987, M.E.Abd El- Monsef et al. [3] introduced the fuzzy supra topological spaces and studied fuzzy supra-continuous functions. This class of sets contained in the class of β-open sets [1] and contains all semiopen sets [2] and all pre-open sets [6]. In 2008, R.Devi et al [5] introduced and studies a class of sets and maps between topological spaces called supra α-open and supra sα-continuous maps, respectively. Now, we introduce the concept of supra I-open set, and supra I-continuous functions and investigate several properties for these classes of maps.

1.1 Definition

A sub family T* of X is said to be a supra topology on X if, (i) $x, \phi \in \tau^*$, (ii) if $A_i \in \tau^*$, $\forall$ i $\in$ j, then $UA_i \in \tau^*$

(x, τ^*) is called supra topological space. The elements of τ^* are called supra open sets in (x, τ^*) and complements of a supra open set is called a supra closed set.

1.2 Definition

The supra closure 0 a set A is denoted by supra cl(A) and defined as supra cl(A) = $\cap$ {B : B is a supra closed and A $\subseteq$ B}. The supra interior of a set A is denoted by Supra int(A), and defined as supra int(A) = $\cup$ {B : B is a τ supra open and A $\supseteq$ B}.

1.3 Definition

Let (x, τ) be a topological space and τ^* be a supra topology on x. We call τ^* a s~~upra topology associated~~ with τ if $\tau \subset \tau^*$.

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2 SUPRA I-OPEN SETS

In this section, we introduce a new class of generalized open sets called supra I-open sets and study some of their properties.

2.1 Definition

Let (x, τ^*) be a supra topological space. A set A is called supra I-open set if A $\subseteq$ Supra int (supra cl(A)). The complement of a supra I-open set is called a supra I-closed set.

2.2 Theorem

Every supra open set is supra I-open set.

Proof.

Let A be a supra open set in (x, τ^*) since, A $\subseteq$ supra cl(A), then supra int(A) $\subseteq$ supra int (supra cl(supra int (A))). Hence

A $\subseteq$ supra int (supra cl (supra int (A))).

The converse of the above theorem need not be true as shown by the following examples.

2.3 Example

Let (x, τ^*) be a supra topological space. Where X = {a, b, c} and τ^* = {x, ϕ, {a}, {a, b}, {b, c}}. Here, {a, c} is a supra I-open set, but it is not a supra open.

2.4 Theorem

In [3], the author proved that every supra I-open set is supra semi-open. (Instead of example 3.2 [3]) shows the converse need not be true.

2.5 Example

Let (x, τ^*) be a supra topological space. Where X = {a, b, c, d} and τ^* = {x, ϕ, {a}, {b}, {a, b}}. Here {b, c} is a supra semi-open set, but not a supra I-open.

2.6 Theorem

(i) Finite union of Supra I-open sets is always a supra I-open.

(ii) Finite intersection of supra I-open sets may fail to be a supra I-open.

Proof:

(i) Let A and B be two supra I-open sets. The $A \subseteq$ supra int (supra cl(supra int (A))) and $B \subseteq$ supra int(supra Cl(supra int (B))). Then $A \cup B \subseteq$ supra int (supra cl(supra int $(A \cup B)$)).

Therefore $A \cup B$ is supra I-open set.

(ii) Let (x, τ^*) be a supra topological space. Where X = {a, b, c, d} and τ^* = {x, ϕ, {a, b}, {b, c}}. Here {a, b}, {b, c} are {a, b}, {b, c} are supra I-open sets but their intersection is a not supra I-open set.

2.7 Theorem

(i) Arbitrary intersection of supra I-closed sets is always supra I-closed.

(ii) Finite union of supra I-closed set may fail to be supra I-closed set.

Proof.

(i) This follows immediately from theorem 2.6.

(ii) Let (x, τ^*) be a supra topological space where x = {a, b, c, d} and τ^* = {x, ϕ, {a, b}, {b, c}} Here {c, d}, {a, d} and supra I-closed sets, but their union is not a supra I-closed set.

2.8 Definition

The supra I-closure of a set A is denote by supra I cl(A) and defined as, supra I cl(A) = $\cap$ {B : B is a supra I-closed set and $A \subseteq B$}.

The supra I-interior of a set is denoted by supra I int(A), and defined as, supra I int(A) = $\cup$ {B : B is a supra I-open set and $A \supseteq B$}.

2.9 Remarks

It is clear that supra I int(A) is a supra I-open set and supra I cl(A) is a supra I-closed set.

2.10 Theorem

(i) X-supra I int(A) = supra I cl(X-A)

(ii) X-supra I cl(A) = supra I int (X-A)

Proof. Obvious.

2.11 Theorem

(i) Supra I int(A) $\cup$ supra I int(B) = supra I int($A \cup B$)

(ii) Supra I cl(A) $\cap$ supra I cl(B) = supra I cl ($A \cap B$)

Proof. Obvious

3 SUPRA I-CONTINUOUS FUNCTIONS

In this section, we introduce a new type of continuous functions called a supra I-continuous function and obtain some of their properties and characterizations.

3.1 Definition

Let (X, τ) and (Y, σ) be two topological space and τ^* be associated supra topology with τ. A function $f : (X, Z), \rightarrow (Y, \sigma)$ is called a supra I-continuous functions if the inverse image of each open set in y is a supra I-open set in x.

3.2 Theorem

Every continuous function is supra I-continuous functions.

Proof.

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a continuous function. Therefore $f^{1}(A)$ is a open set in x for each open set A in Y. But, τ^* is associated with τ. That is $\tau \subset \tau^*$. This implies $f^{1}(A)$ is a supra open in x. since supra open is supra I-open this implies $f^{1}(A)$ is supra I-open in X. Hence f is a I continuous functions.

The converse of the above theorem is not true as shown in the following example.

3.3 Example

Let X = {a, b, c} and τ = {x, ϕ, {a, b}} be a topology on X. The supra topology τ^* is defined as follows, τ^* = {x, ϕ, {a}, {a, b}}. Let $f : (X, \tau) \rightarrow (X, \tau)$ be a function defined as follows $f(a) = a$, $f(b) = c$, $f(c) = b$. The inverse image of the open set {a, b} is {a, c} which is not an open set but if a supra I-open. Then f is supra I-continuous but it is not continuous.

3.4 Theorem

Let (X, τ) and (Y, σ) be two topological spaces. Let f be a function from X
into Y. Let τ^* be associated supra topology with τ. Then the following are equivalent.

a) f is I-continuous.
b) The inverse image of closed set in Y in supra I-closed set in X.
c) Supra I cl(f^{1}(A)) $\subseteq f^{1}$(cl(A)) for every set A in Y.
d) f (supra I cl(A)) $\subseteq$ cl(f(A)) for every set A in X.
e) f^{1}(int(B)) $\subseteq$ supra I int(f^{1}(B)) for every set B in Y.

Proof.

(a) $\Rightarrow$ (b) : Let A be a closed set in Y, then Y-A is open in Y. Thus,
f^{1} (X-A) = X – f^{-1}(A) in supra I-open in X. It follows that f^{1}(A) in a supra I-closed set of X.

(b) $\Rightarrow$ (c) : Let A be any subset of X. Since cl(A) is closed in Y, then it follows that f^{1}(cl(A)) is supra I-closed in X. Therefore f^{1}(cl(A)) = supra I cl(f^{1}(cl(A)) $\supseteq$ supra I cl(f^{1}(A)).

(c) $\Rightarrow$ (d) : Let A be any subset of X. By (c) we obtain, f^{1} (cl (f(A))) $\supseteq$ supra I cl(f^{1} (A))) $\supseteq$ supra I cl(A) and hence f(supra I cl(A)) $\subseteq$ cl(f(A)).

(d) $\Rightarrow$ (e) : Let f (supra I cl(A)) $\subseteq$ cl (f(A)) for every set A in X. Then supra I cl (A) $\subseteq f^{1}$ (cl(f(A))), X-supra I cl(A) $\supseteq$ X – f^{1}(cl (f (A))) and supra
I int (X – A) $\supseteq f^{1}$ (int (y-f(A))). Then supra I int (f^{1}(B)) $\supseteq f^{1}$(int (B)). Therefore f^{1}(int(B)) $\subseteq$ supra I int (f^{1} (B)), for every B in Y.

(e) $\Rightarrow$ (a) : Let A be a open set in Y. Therefore f^{1} (int (A)) $\subseteq$ supra
I int (f^{1} (A)), hence f^{1} (A) $\subseteq$ supra I int (f^{1}(A)). But by other hand we know that, supra I int(f^{1}(A)) $\subseteq f^{1}$(A). Then f^{1}(A) = supra I int (f^{1} (A)). Therefore, f^{1} (A) is a supra I-open set.

3.5 Theorem

If a map f : (X, τ) $\rightarrow$ (Y, σ) is supra I-continuous and g : (Y, σ) $\rightarrow$ (Z, γ) is continuous map, then g.f : (X, τ) $\rightarrow$ (Z, γ) is supra I-continuous.

Proof. Obvious.

3.6 Theorem

Let (x, τ) and (y, σ) be topological space. Let τ^{*} and σ^{*} be associated supra topologies with τ and σ respectively. Then f : (x, τ) $\rightarrow$ (y, σ) is supra I-continuous map, if one of the following holds.

(1) f^{1} (supra I Int(B)) $\subseteq$ int (f^{1} (B) for every set B in Y.
(2) cl (f^{1} (B)) $\subseteq f^{1}$ (supra I cl(B)) for every set B in Y.
(3) f (cl (A)) $\subseteq$ supra-I cl (f (A)) for any set A in X.

Proof.

Let B be any open set of Y, if condition (1) is satisfied, then f^{1} (supra I int (B)) $\subseteq$ int (f^{1} (B)). We get f^{1} (B) $\subseteq$ int (f^{1} (B)). Therefore f^{1}(B) is supra open set. Every supra open set is supra I-open set. Hence f is a supra I-continuous map.

If condition (2) is satisfied, then we can easily prove that f is a supra I-continuous map.

If condition (3) is satisfied and B is any open set of Y. then f^{1}(B) is a set in
X and f(cl(f^{1} (B))) $\subseteq$ supra I cl(f(f^{1} (B)). This implies f(cl (f^{1} (B))) $\subseteq$ supra I cl(A). This is nothing but conditions (2). Hence f is a supra I-continuous map.

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